Tentamen Quantum Fysica I

June 28, 1999

Please **print** *your name, student number and complete address on the first page. Each problem is to be answered on a* separate *page. Print your name on top of each page.*

Elke opgave op en apart vel. Zet op het eerste vel duidelijk uw naam, student nummer en adres. Op elk volgend vel uw naam vermelden.

Problem 1.

(a) For an operator A, under what circumstance is it Hermitian (i.e. $A = A^{+}$) ?

(b) Write the Time Dependent Schrödinger Equation. What is meant by a "stationary state"?

(c) What is meant by the "Schrödinger picture" and by the "Heisenberg picture" ?

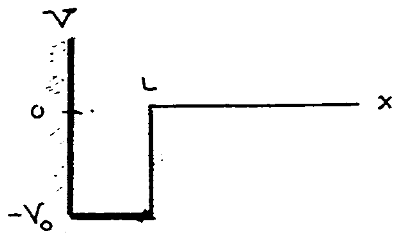
(d) Which of the following pairs of operators can have simultaneous eigenfunctions? Explain.

  (i) p and $T = p^2/2m$

  (ii) p and $V(x)$

Problem 2.

A particle of mass m is confined to move in one dimension by a potential $V(x)$



$$V(x) = \begin{cases} +\infty & x < 0 \\ -V_0 & 0 < x < L \\ 0 & L < x \end{cases}$$

(a) Derive an equation for the bound state.

(b) Derive an expression for the magnitude V_0 for which there will be only *one* bound state, and that its energy is $E = -V_0/2$.

Problem 3.

A particle moving in one dimension in an infinite square-well has as its initial ($t = 0$) wave function the sum of its first two normalized eigenfunctions ($u_1(x)$ whose eigenvalue is E_1, and $u_2(x)$ whose eigenvalue is E_2):

$$\psi(x, 0) = A[u_1(x) + u_2(x)].$$

In parts (a) - (g) not *necessary to use the specific functional form of the* $u_n(x)$ *nor do you need to evaluate integrals involving them. Many of the questions below are independent of each other. If you get stuck on one, then keep on going!*

(a) Normalize $\psi(x, 0)$, *i.e.*, find A.

(b) Find $\psi(x, t)$ and $|\psi(x, t)|^2$.

(c) Are the eigenfunctions $u_1(x)$ and $u_2(x)$ eigenstates of the parity operator?

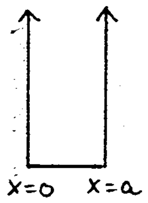
(d) Find, and reduce to the simplest possible form, an expression for the expectation value of the particle position, $< x > = < \psi|x|\psi >$, as a function of time, for the state $\psi(x, t)$ determined in part (b). (Hint: See (c))

(e) Repeat (d) for $< p > = < \psi|p|\psi >$.

(f) State why, in spite of the fact that $\psi(x, t)$ can be complex, the $< x >$ and $< p >$ evaluated in (d) and (e) are real.

(g) Find the expectation value of H. How does it compare with E_1 and E_2?

Suppose that the infinite-square-well extends from $x = 0$ to $x = a$.



(h) Determine E_1 and E_2 in terms of the mass of the particle m, $\hbar$ and a.

(i) A *classical* particle in this well would bounce back and forth between the walls. If its energy is equal to the expectation value that you found in (g), what is the frequency of the classical motion? Compare with the quantum frequency you found in (d).